

Commentary on the Habilitation thesis

Boundary value problems for nonlinear elliptic equations with a Hardy potential

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The habilitation thesis presents recent developments on boundary value problems for nonlinear elliptic equations with a Hardy potential in a measure framework. The interplay between the Hardy potential and measures characterizes substantial difficulties in the analysis and reveals the novelty of the research. New aspects are displayed not only on employed methods but also on observed novel phenomena.

The thesis comprises five chapters. The first chapter addresses the main topics covered in the thesis and presents our contributions, which are collected from our recent works [3, 4, 1, 2], including results on the existence, nonexistence, uniqueness, a priori estimates and qualitative properties of solutions, a full characterization of isolated boundary singularities, removable singularities. The major features of the problems under investigation depend essentially on the expression of nonlinear terms in equations. Therefore, typical models are successively considered throughout these chapters in order to reveal different phenomena. In particular, chapter 2 treats absorption nonlinear terms and chapter 3 is concerned with source nonlinear terms. Striking distinctions regarding the existence and uniqueness between the absorption case and source case are pointed out. Chapter 4 is devoted to an extension of results in the previous chapters to more general equations and systems. Finally chapter 5 focuses on the case where nonlinear terms depend on both solutions and their gradient.

The above results are based on my joint paper with Moshe Marcus [3], with Konstantinos Gkikas [1, 2] and my single-author paper [4], which have been published in highly ranked journals. These papers are the outcome of fruitful collaborations during which ideas have been shared and a lot of discussions have been carried out. Therefore, the contributions of all collaborators are meant to be equivalent. In this regard, my contribution to the joint papers [1, 2, 3] is 50%.

References

- [1] K. T. GKIKAS AND P. T. NGUYEN, *On the existence of weak solutions of semilinear elliptic equations and systems with Hardy potentials*, J. Differential Equations **266** (2019), no. 1, 833–875.

- [2] K. T. GKIKAS AND P. T. NGUYEN, *Elliptic equations with Hardy potential and gradient-dependent nonlinearity*, Adv. Nonlinear Stud. **20** (2020), no. 2, 399-435.
- [3] M. MARCUS AND P.-T. NGUYEN, *Moderate solutions of semilinear elliptic equations with Hardy potential*, Ann. Inst. H. Poincaré Anal. Non Linéaire **34** (2017), no. 1, 69-88.
- [4] P.-T. NGUYEN, *Semilinear elliptic equations with Hardy potential and subcritical source term*, Calc. Var. Partial Differential Equations **56** (2017), no. 2, Paper No. 44, 28 pp.